

BUTT SPLICE HINGING

BY

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Introduction

Splicing is a process used to join the tail of an expiring roll to the start of a new roll so that web handling processes can continue to operate uninterrupted. Depending on the needs of the process, the ends of the webs can either be overlapped or positioned adjacent to each other and then, in most applications, joined using tape. The first is referred to as lap splicing and the second as butt splicing. While lap splicing is operationally easiest to implement, it is not the more robust of the two in terms of interactions with many types of coating processes. Butt splicing is preferred because it eliminates web tails and, when constructed to appropriate tolerances, adequately mitigates coating disturbances as the splice passes through the coating process. In spite of this advantage, coating processes are not completely transparent to butt splices since coating typically takes place as the web conveys over rollers. In this case, butt splices have the negative tendency to lift off, or *hinge*, as they convey over the coating roller. This behavior is a consequence of the discontinuity in out-of-plane bending stiffness in the vicinity of the splice due to the tape and the gap between the tail of the expiring roll and the start of the new roll. In this paper, a model is described that predicts the magnitude of butt splice hinging as a function of process and product variables. Experimental measurements are correlated to model predictions and the model is then used to demonstrate the relative effects of the parameters on splice hinging. Design guidelines to minimize the magnitude of butt splice hinging are also presented.

Splice Geometry and Model Assumptions

One of the objectives of this paper is to develop a model to predict the magnitude of splice hinging that takes place as a butt splice transitions around a roller. To develop this model, the following assumptions have been made: (a) the constitutive behavior of the web and splice tape are governed by linear elasticity, (b) the deflection of the web and splice tape are described by Euler beam deflection theory, and (c) the effect of tension stiffening is included in the formulation of the differential equations describing the web and tape deflections. Not included in the model are the effects of speed and air entrainment.

Figure 1 shows a cross sectional view of web under tension, T , transitioning around a roller of radius R . For convenience, the effect of the splice lifting off of the roller is shown midway around the wrap of the roller. In this analysis, the wrap angle is assumed to be adequately large such that the free spans upstream and downstream of the roller do not affect the splice hinging area.

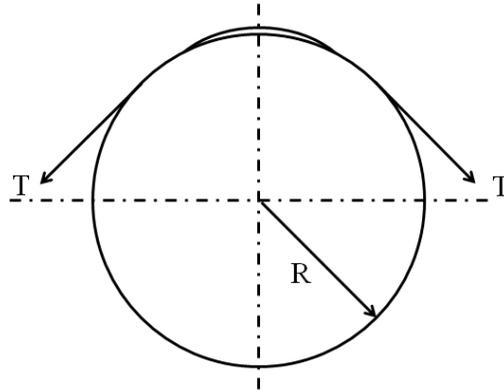


Figure 1: Roller/Web Geometry

Figure 2 shows a more detailed schematic of the area affected by the splice. The origin of the coordinate system about which deflections are computed is taken to be at the point where the web peels away from the roller due to hinging and is oriented to align with the neutral axis of the composite web/tape beam. Additional variables are web thickness, t_1 , web modulus, E_1 , tape thickness, t_2 , tape modulus, E_2 and half gap width, L_2 . The unknowns are the splice hinging, c , and peel length, L_1 .

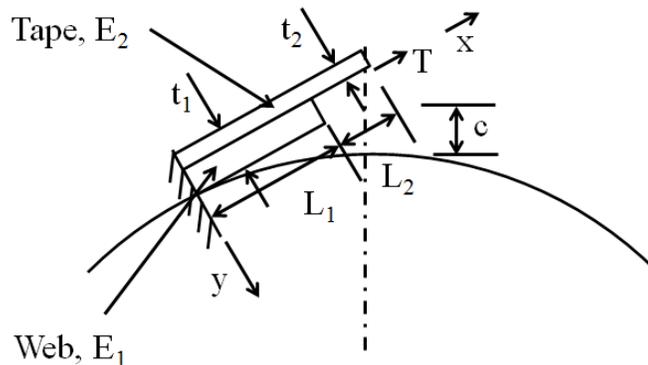


Figure 2: Splice Geometry

The differential equations used to model the deflection of the splice are developed following the sign convention shown in Figure 3. From this, the following relationships between moment, shear and distributed load are obtained (where the bending stiffness $I = \frac{1}{12} t^3$):

$$M = -EI \frac{d^2 y}{dx^2} \quad (1)$$

$$N = -EI \frac{d^3 y}{dx^3} \quad (2)$$

$$w(x) = EI \frac{d^4 y}{dx^4} - T \frac{d^2 y}{dx^2} \quad (3)$$

In the free span, the distributed load equals zero and the differential equation governing the deflection can therefore be written as:

$$\frac{d^4 y}{dx^4} - \kappa^2 \frac{d^2 y}{dx^2} = 0 \quad (4)$$

where $\kappa = \sqrt{\frac{T}{EI}}$. Equation (4) can now be applied to each section of the splice where section 1 is

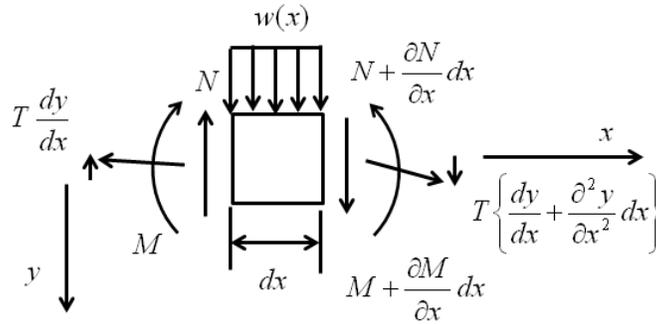


Figure 3: Sign Conventions

the composite web/tape and section 2 is the tape only:

$$\frac{d^4 y_1}{dx^4} - \kappa_1^2 \frac{d^2 y_1}{dx^2} = 0 \quad (5)$$

and

$$\frac{d^4 y_2}{dx^4} - \kappa_2^2 \frac{d^2 y_2}{dx^2} = 0 \quad (6)$$

where $\kappa_1 = \sqrt{\frac{T}{E_1 I_{1e}}}$ and $\kappa_2 = \sqrt{\frac{T}{E_2 I_2}}$. For section 2, E and I are for the tape only while for section 1, consideration must be given to the fact that we have a composite beam. Using the principle of a *transformed section* (Strength of Materials, Timoshenko, page 218), the deflection of section 1 can be determined by computing the flexural stiffness of section 1 where the tape is replaced by an equivalent material of modulus E_1 . To achieve equivalent deflection, the width of the tape is adjusted as shown in Figure 4. Referring to the figure, the neutral axis can be found from the following (based on the definition that the neutral axis is the location about which the first moment of area is equal to zero):

$$d = \frac{(t_1 - t_2)^2 + t_2(2t_1 - t_2)\frac{E_2}{E_1}}{2\left\{t_1 - t_2\right\} + \frac{E_2}{E_1}t_2} \quad (7)$$

The bending stiffness for section 1 can now be computed using the parallel axis theorem:

$$I_{1e} = \frac{1}{12}(t_1 - t_2)^3 + (t_1 - t_2)\left(d - \frac{t_1 - t_2}{2}\right)^2 + \frac{1}{12}\frac{E_2}{E_1}t_2^3 + t_2\frac{E_2}{E_1}\left(d - \frac{2t_1 - t_2}{2}\right)^2 \quad (8)$$

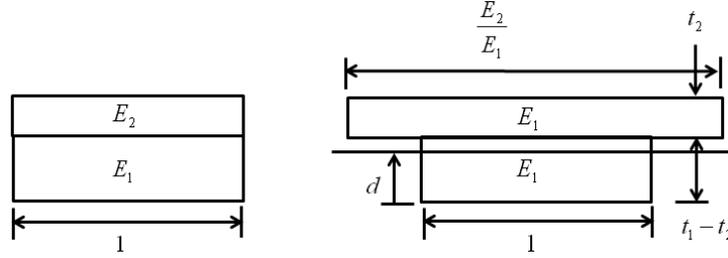


Figure 4: Transformed Section 1

General Solutions and Boundary Conditions

The general solution to equations (5) and (6) are given by:

$$y_1(x) = c_1 \sinh(\kappa_1 x) + c_2 \cosh(\kappa_1 x) + c_3 x + c_4 \quad (9)$$

$$y_2(x) = c_5 \sinh(\kappa_2 x) + c_6 \cosh(\kappa_2 x) + c_7 x + c_8 \quad (10)$$

Nine conditions are required to achieve a solution to the problem since there are nine unknowns (eight boundary conditions and the peel length). Three can be written at $x = 0$:

$$y_1(0) = 0 ; \quad \text{displacement equals zero} \quad (11)$$

$$\frac{dy_1(0)}{dx} = 0 ; \quad \text{slope equals zero} \quad (12)$$

$$\frac{d^2 y_1(0)}{dx^2} = \frac{1}{R} ; \quad \text{curvature equals the inverse of the roller radius} \quad (13)$$

Four additional equations can be written at the intersection between section 1 and section 2:

$$y_1(L_1) = y_2(L_1) ; \quad \text{displacement match at boundary} \quad (14)$$

$$\frac{dy_1(L_1)}{dx} = \frac{dy_2(L_1)}{dx} ; \quad \text{slope match at boundary} \quad (15)$$

$$E_1 I_{1e} \frac{d^2 y_1(L_1)}{dx^2} - T \left\{ (t_1 - d) - \frac{t_2}{2} \right\} = E_2 I_2 \frac{d^2 y_2(L_1)}{dx^2} ; \quad \text{moment jump at boundary} \quad (16)$$

$$E_1 I_1 e \frac{d^3 y_1(L_1)}{dx^3} = E_2 I_2 \frac{d^3 y_2(L_1)}{dx^3} ; \quad \text{shear match at boundary} \quad (17)$$

The final two boundary conditions are found from the boundary at the end of section 2. Since this is plane of symmetry, the slope is a known function of the geometry and the shear is equal to zero:

$$\frac{dy_2(L_1+L_2)}{dx} = \frac{L_1+L_2}{R} ; \quad \text{slope is related to position on the roller} \quad (18)$$

$$\frac{d^3 y_2(L_1+L_2)}{dx^3} = 0 ; \quad \text{shear equals zero at plane of symmetry} \quad (19)$$

The nine boundary conditions were used to develop a transcendental equation for the determination of L_J . This equation was programmed into Matlab™ and the resulting model used to generate the results of the next section.

Modeling Results and Discussion

The model was used to study the effects of each of the following variables on the prediction of clearance due to hinging, c , and peel length, L_J :

- web modulus
- tape modulus
- web thickness
- tape thickness
- half gap width
- roller radius
- tension

For each variable, three levels were studied while holding each of the remaining variables constant. The nominal values for each are as follows:

- web modulus: 6.00e5 psi – nominal, (5.00e5, 6.00e5, 7.00e5 range)
- tape modulus: 6.00e5 psi – nominal, (5.00e5, 6.00e5, 7.00e5 range)
- web thickness: 0.005 inch – nominal, (0.004, 0.005, 0.006 range)
- tape thickness: 0.002 inch – nominal, (0.002, 0.003, 0.004 range)
- half gap width: 0.0469 inch – nominal, (0.025, 0.050, 0.075 range)
- roller radius: 2 inch – nominal, (1, 2, 3 range)
- tension: 0.1 to 1 pli

Figures 5 through 16 show the results of each of these effects. Also indicated on each drawing are experimental results obtained at three tension levels. A noncontacting laser sensor was used to measure clearance and peel length. The characteristics for which the measurements were obtained are as follows: (a) web modulus – 6.00e5 psi, (b) tape modulus – 6.00e5 psi, (c) web

thickness – 0.005 inch, (d) tape thickness – 0.002 inch, (e) half gap width – 0.050 inch, and (f) roller radius – 2 inch. Measurements were made at three levels of tension – 0.37 pli, 0.56 pli and 0.74 pli.

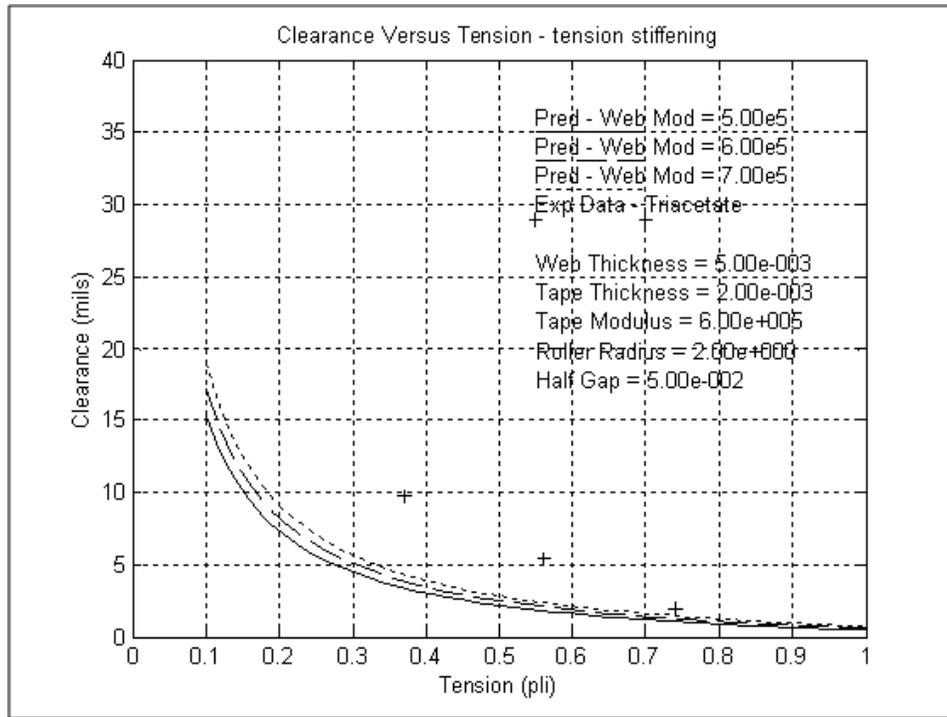


Figure 5: Clearance vs. Tension - Web Modulus

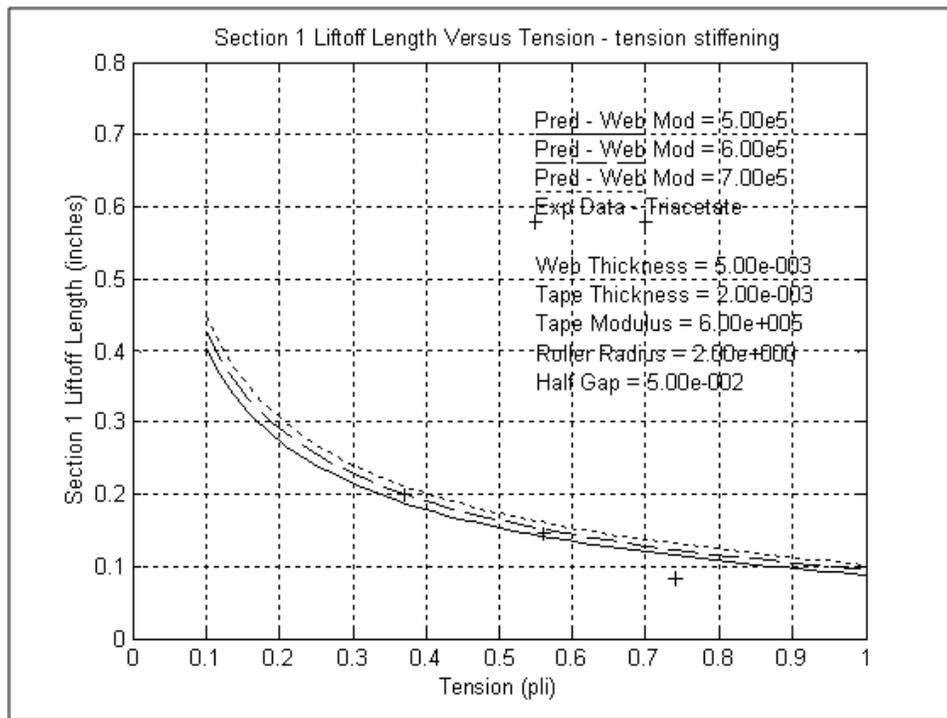


Figure 6: Peel Length vs. Tension - Web Modulus

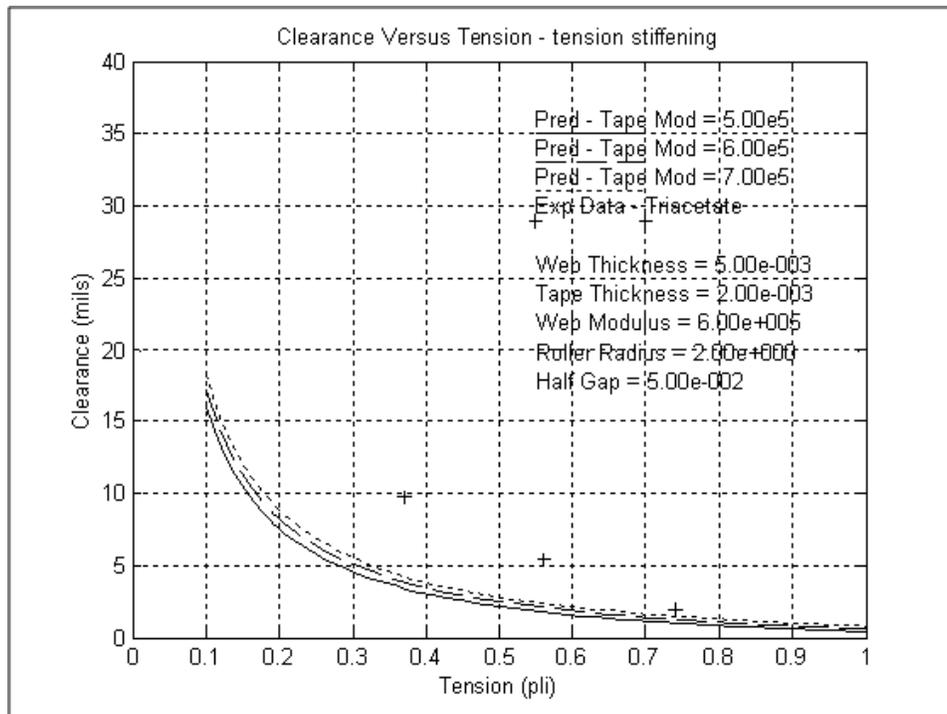


Figure 7: Clearance vs. Tension - Tape Modulus

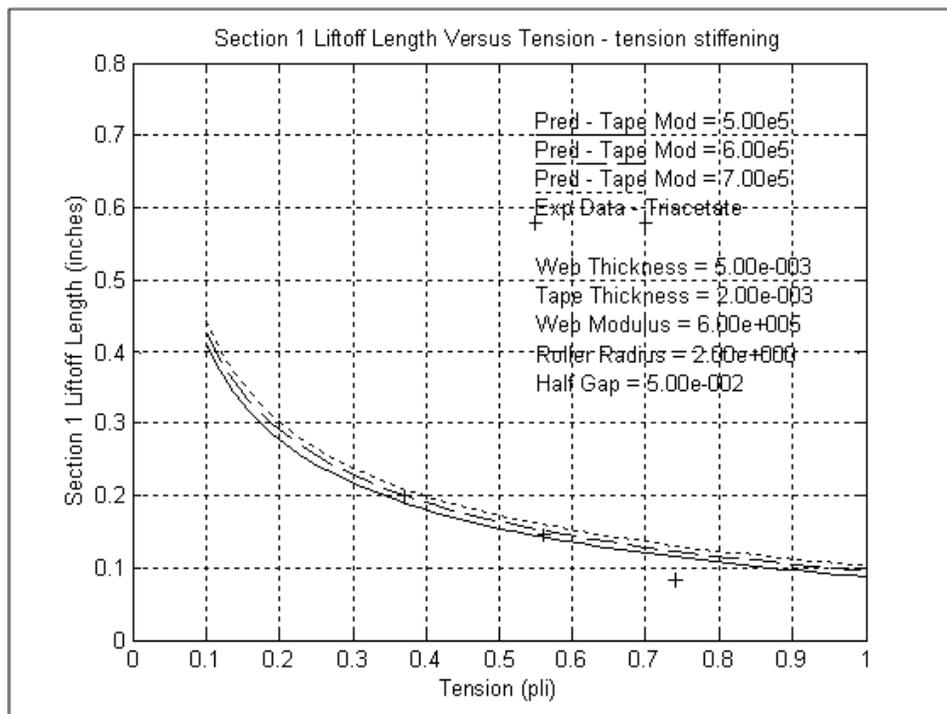


Figure 8: Peel Length vs. Tension - Tape Modulus

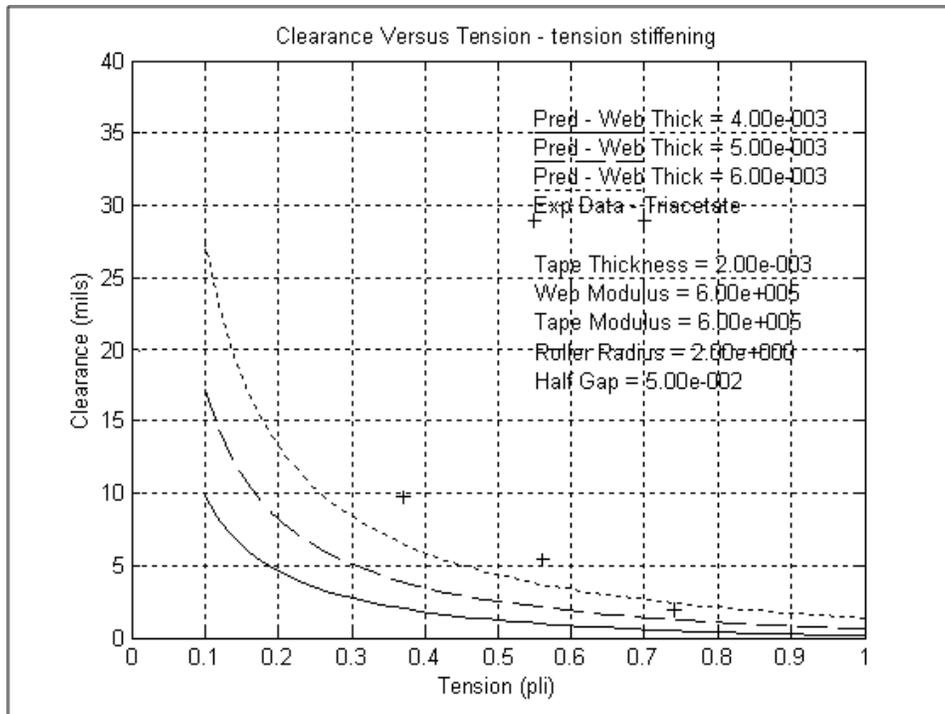


Figure 9: Clearance vs. Tension - Web Thickness

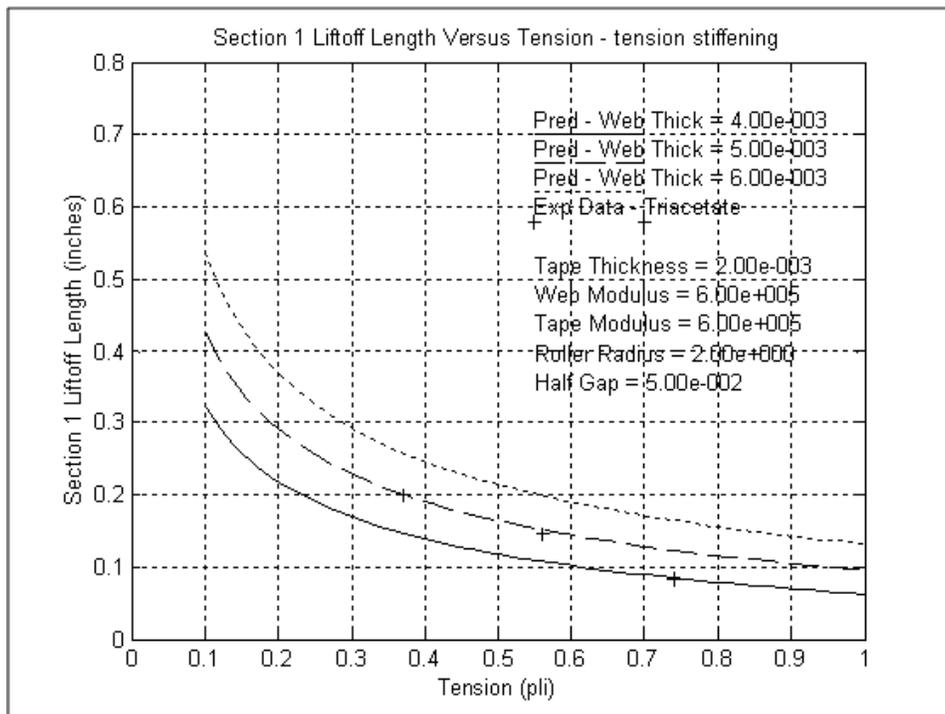


Figure 10: Peel Length vs. Tension - Web Thickness

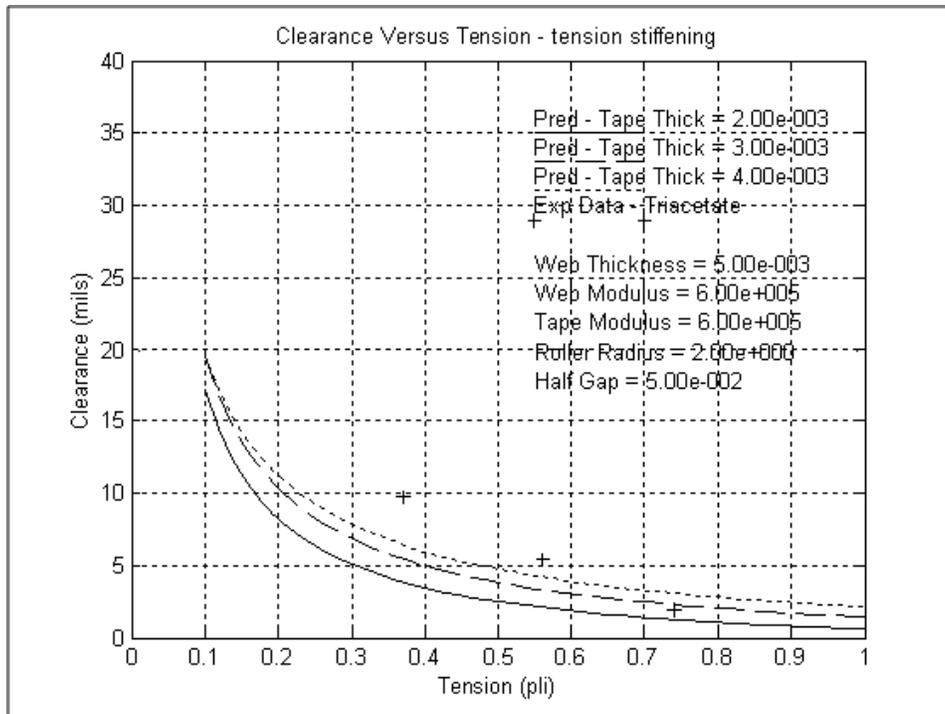


Figure 11: Clearance vs. Tension - Tape Thickness

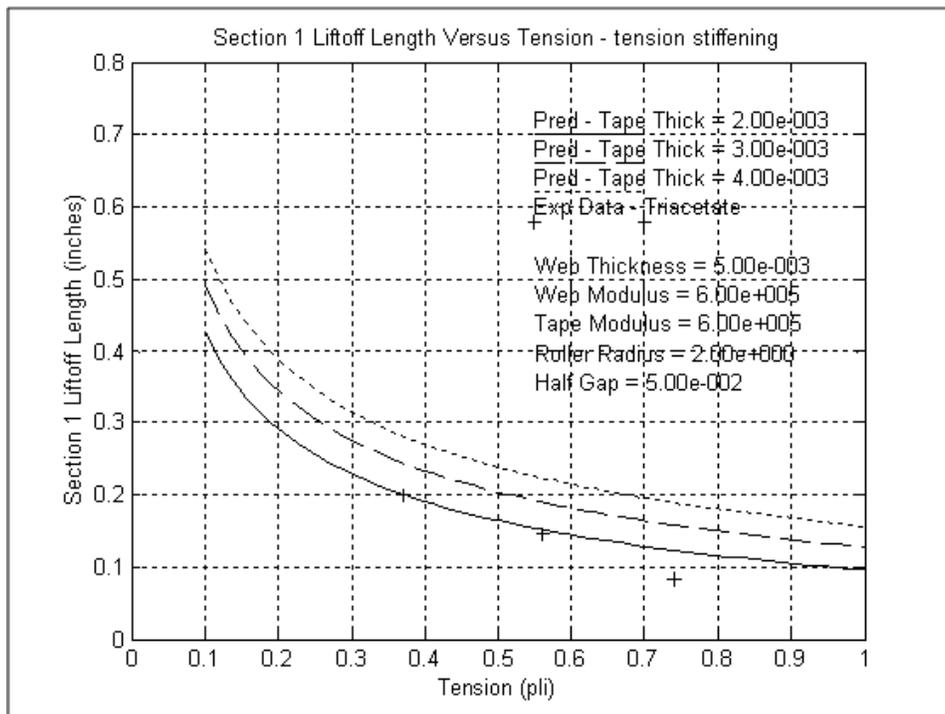


Figure 12: Peel Length vs. Tension - Tape Thickness

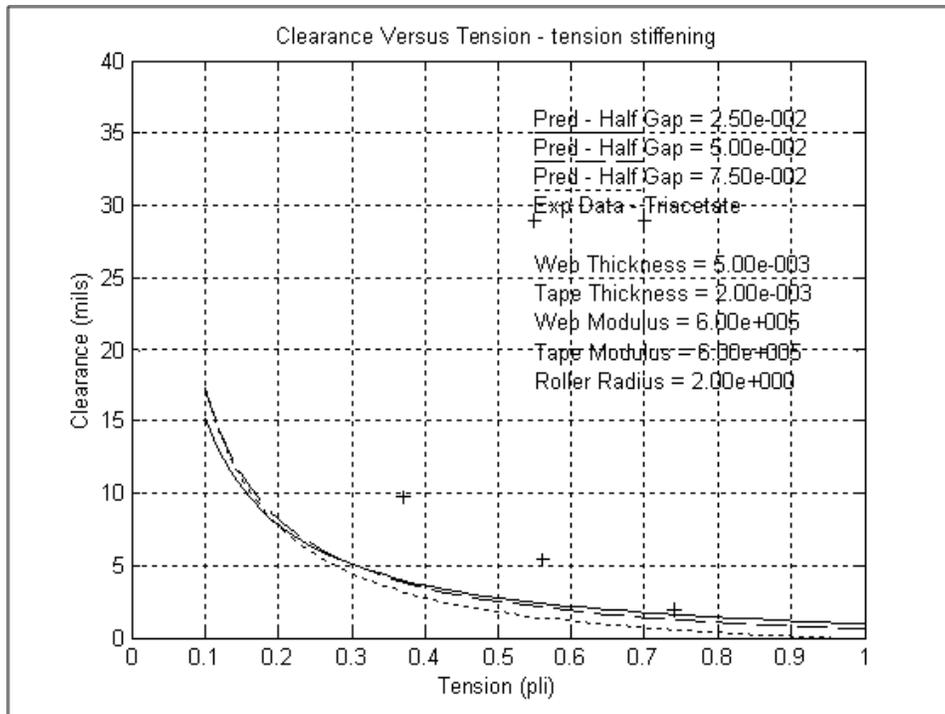


Figure 13: Clearance vs. Tension - Half Gap Width

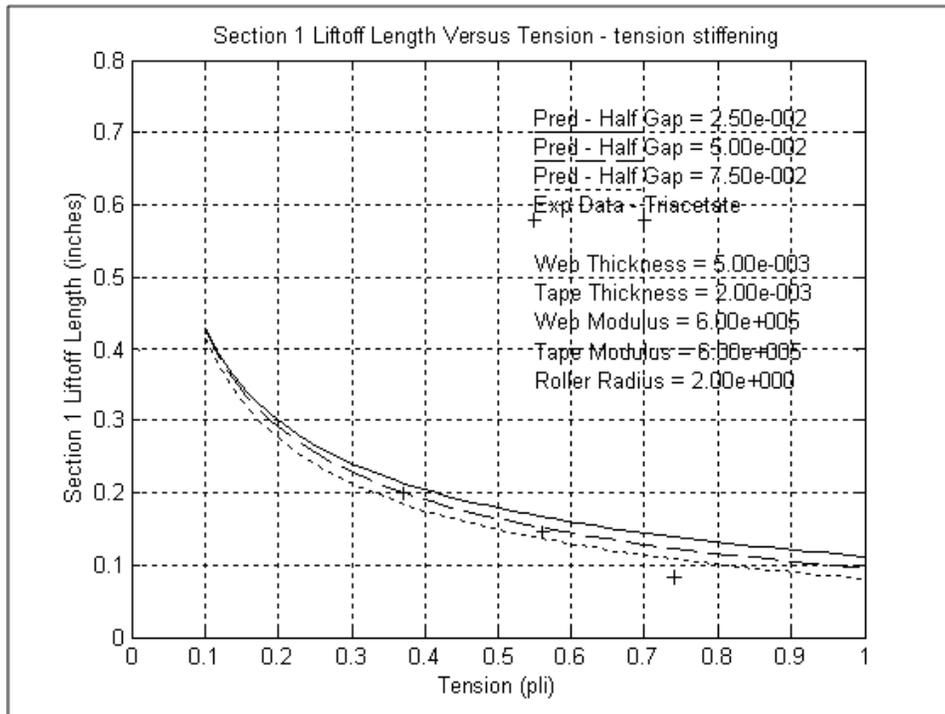


Figure 14: Peel Length vs. Tension - Half Gap Width

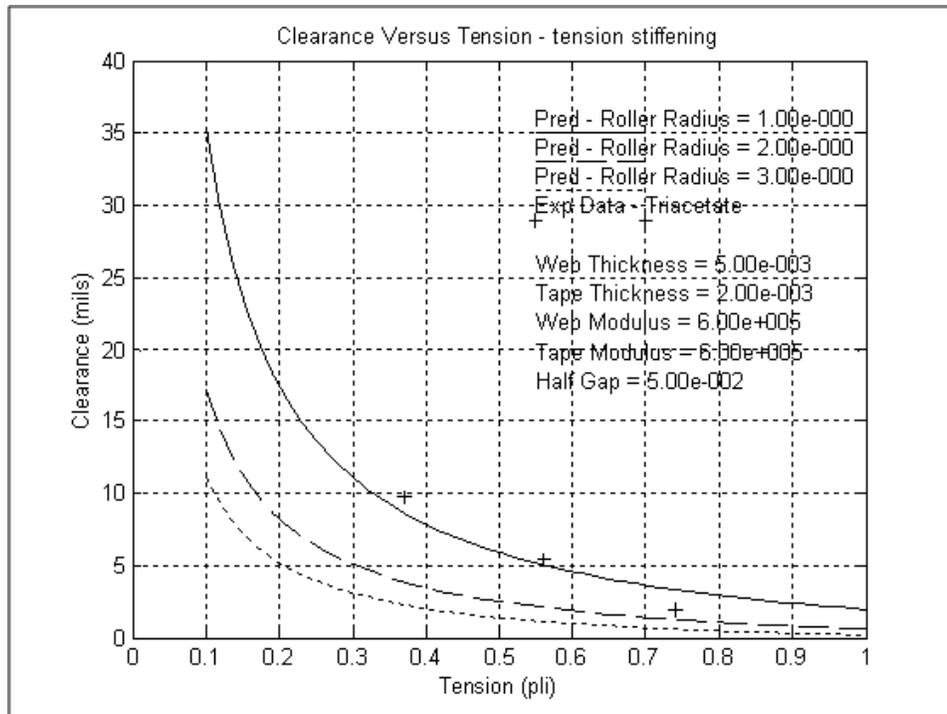


Figure 15: Clearance vs. Tension - Roller Radius

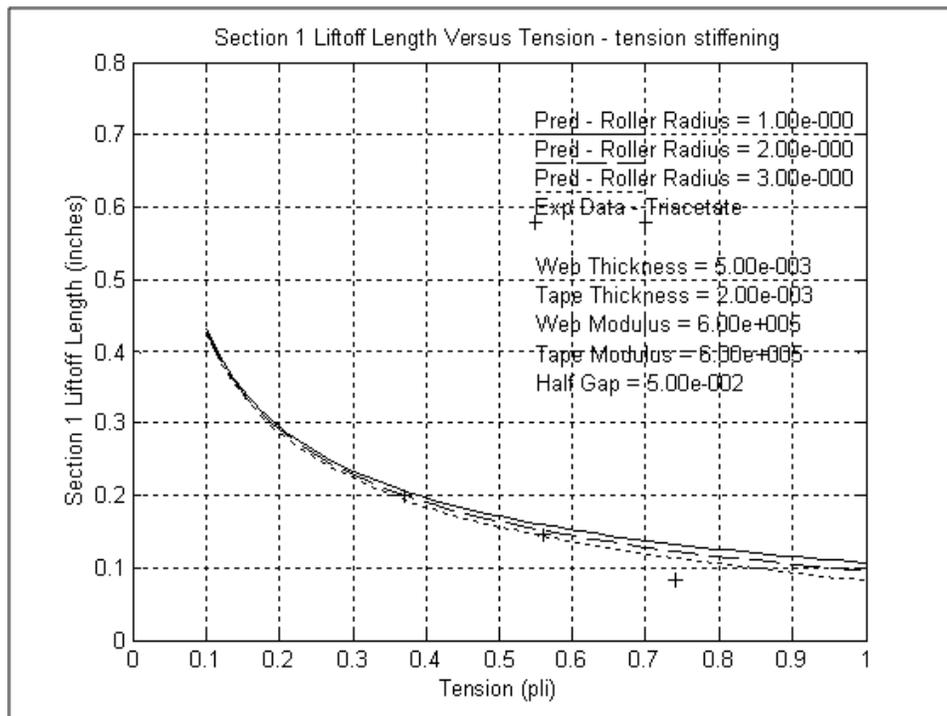


Figure 16: Peel Length vs. Tension - Roller Radius

Observation of Figures 5 through 16 indicates the following trends:

- Main effects on hinging clearance – web thickness, roller radius, tension

- Secondary effects on hinging clearance – web modulus, tape modulus, tape thickness, half gap width
- Main effects on peel length – web thickness, tape thickness, tension
- Secondary effects on peel length – web modulus, tape modulus, half gap width, roller radius

Table 1 summarizes these observations and also indicates the directional impact of an increase in the variable on the response.

Variable	Hinging	Direction	Peel Length	Direction
Web Modulus	Secondary	Increasing	Secondary	Increasing
Tape Modulus	Secondary	Increasing	Secondary	Increasing
Web Thickness	Main	Increasing	Main	Increasing
Tape Thickness	Secondary	Increasing	Main	Increasing
Half Gap Width	Secondary	Mixed	Secondary	Decreasing
Roller Radius	Main	Decreasing	Secondary	Decreasing
Tension	Main	Decreasing	Main	Decreasing

Table 1: Effect Strength and Direction of Variables on Responses

Further insight into the behavior of the splice can be developed by considering a modification to the analysis of the problem. Towards this end, it is informative to reanalyze the problem by neglecting tension stiffening in section 2. This will result in a conservative estimate of hinging and peel length since the effect of tension stiffening in section 2 will result in a reduction of stiffness meaning that hinging and peel length will increase. In this case, the differential equation governing the deflection in section 2 becomes:

$$E_2 I_2 \frac{d^4 y_2}{dx^4} = 0 \quad (20)$$

The general solution to this equation is:

$$y_2(x) = c_5 x^3 + c_6 x^2 + c_7 x + c_8 \quad (21)$$

which replaces equation (10). Substitution of the boundary conditions into equations (9) and (21), after much algebra, yields the following transcendental equation for the determination of the peel length:

$$\left\{ \frac{E_1 I_1 e}{E_2 I_2} \frac{1}{\cosh \kappa_1} - 1 \right\} L_2 - \frac{TR}{2E_2 I_2} \left\{ (t_1 - d) - \frac{t_2}{2} \right\} L_2 + \left\{ \frac{\tanh \kappa_1}{\kappa_1} - 1 \right\} L_1 = 0 \quad (22)$$

Equation (22) can be simplified by series expansion of the hyperbolic functions to order (2) as follows:

$$\cosh k_1 = \cosh \sqrt{\frac{T}{E_1 I_{1e}}} L_1 \cong 1 + \frac{T}{2E_1 I_{1e}} L_1^2 \quad (23)$$

$$\tanh k_1 = \tanh \sqrt{\frac{T}{E_1 I_{1e}}} L_1 \cong \sqrt{\frac{T}{E_1 I_{1e}}} L_1 - \frac{1}{3} \sqrt{\frac{T}{E_1 I_{1e}}} \frac{T}{E_1 I_{1e}} L_1^3 \quad (24)$$

Substitution of equations (23) and (24) into equation (22) yields the following cubic equation for the determination of the peel length:

$$L_1^3 + \frac{3L_2}{2} \frac{E_1 I_{1e}}{E_2 I_2} L_1^2 - \frac{3L_2 E_1 I_{1e}}{T} \left\{ \frac{E_1 I_{1e}}{E_2 I_2} - 1 - \frac{TR}{2E_2 I_2} \left((t_1 - d) - \frac{t_2}{2} \right) \right\} = 0 \quad (25)$$

As the peel length approaches zero, equation (25) is sufficiently accurate to be used for its determination. Of greater importance is the use of equation (25) to determine the relationship between the variables where a nonzero solution no longer exists. Observation of equation (25) indicates that this will be the case when the bracketed portion of the third term becomes negative. We therefore have the following criteria, assuming no tension stiffening in section 2, for the relationship between the variables where hinging will not occur:

$$\frac{E_1 I_{1e}}{E_2 I_2} - 1 - \frac{TR}{2E_2 I_2} \left((t_1 - d) - \frac{t_2}{2} \right) < 0, \text{ no hinging will occur} \quad (26)$$

or rewriting, we have the following:

$$\frac{TR}{2} \left((t_1 - d) - \frac{t_2}{2} \right) > E_1 I_{1e} - E_2 I_2, \text{ no hinging will occur} \quad (27)$$

Equation (27) indicates that there are critical values of tension and radius above which hinging will not occur. Equation (27) can be further simplified for the case where the moduli of the web and tape are equal:

$$\frac{TR}{E} > \frac{1}{6} \frac{t_1^3 - t_2^3}{t_1 + t_2}, \text{ no hinging will occur} \quad (28)$$

Equation (28) gives the further insight that decreasing modulus leads to an increased likelihood of no hinging as well as a reduction in web thickness, $t_1 - t_2$. Figures (17) through (19) present results from the modified model for the three levels of web modulus shown in Figures (5) and (6). Tension now ranges up to 6 pli. Figures (20) and (21) are corresponding plots from the tension stiffening model plotted up to 6 pli. From these figures, we see that first, the results from the no-tension stiffening model are very conservative and second, that the no-hinging criteria, equation (26) and plotted in Figure (19), accurately predicts the condition where hinging will not take place compared to the detailed non-tension stiffening results shown in Figures (17) and (18).

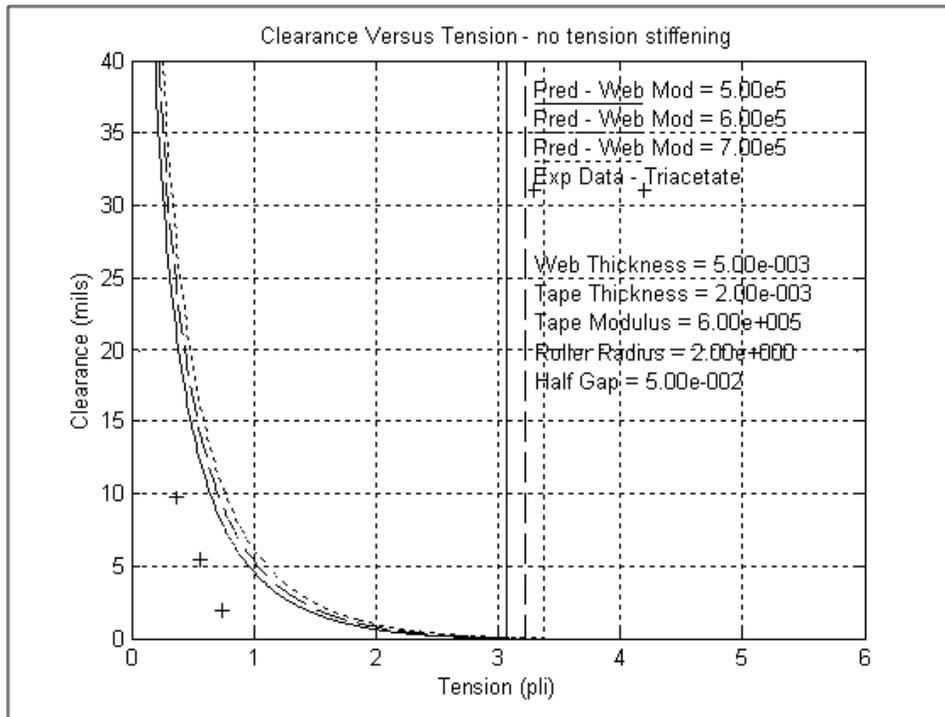


Figure 17: Clearance vs. Tension - Web Modulus, No Tension Stiffening

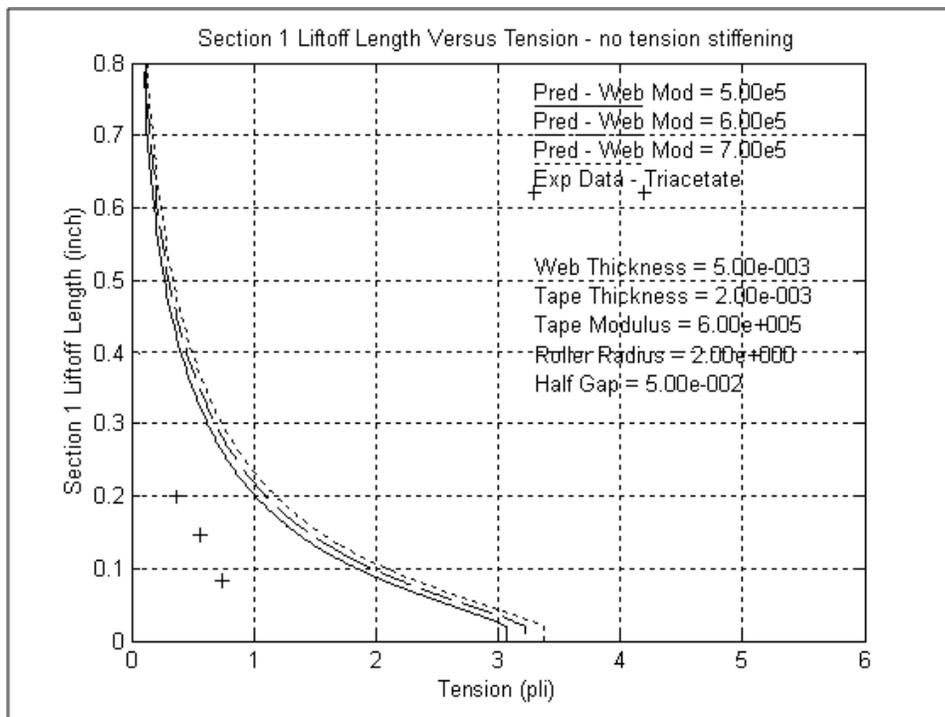


Figure 18: Peel Length vs. Tension - Web Modulus, No Tension Stiffening

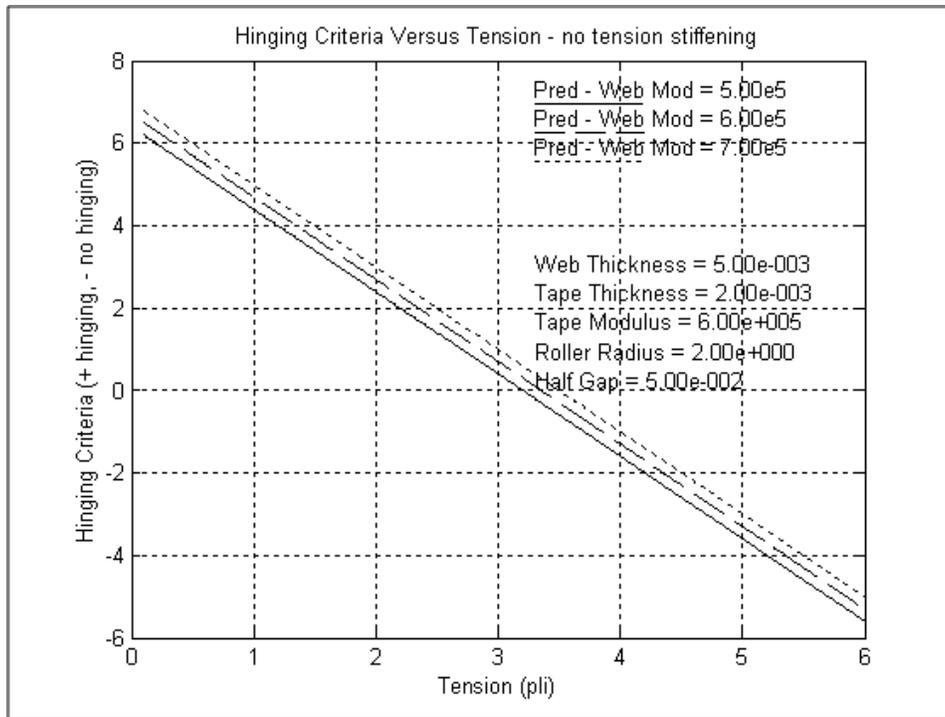


Figure 19: Hinging Criteria, Equation (26) - No Tension Stiffening

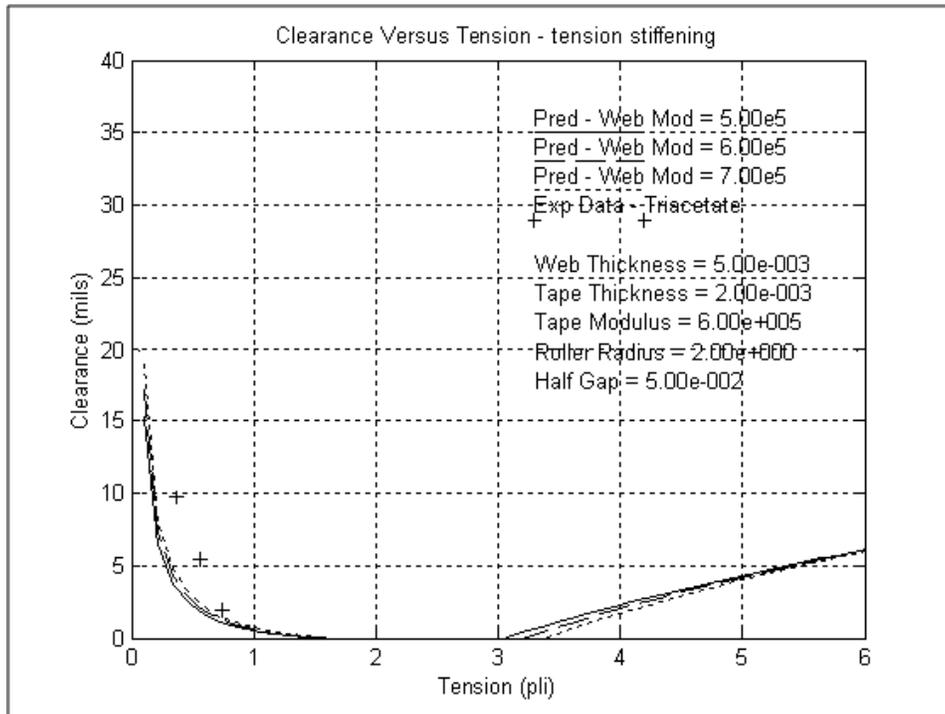


Figure 20: Clearance vs. Tension - Web Modulus, Tension Stiffening

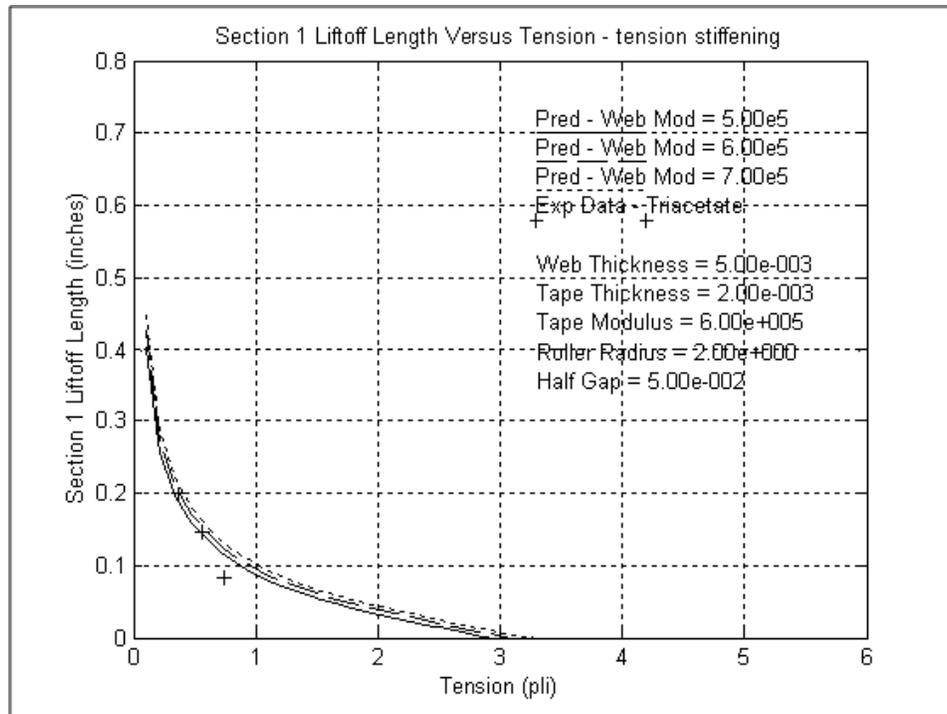


Figure 21: Peel Length vs. Tension - Web Modulus, Tension Stiffening

Conclusions

A model has been presented that predicts splice hinging and peel length for a butt splice conveying over a roller. The butt splice is formed by tape overlapping the tail and lead edge of a web. The model treats the web as an Euler beam with tension stiffening included. The effects of web and tape modulus, web and tape thickness, half gap width, roller radius and tension on hinging and peel length were studied using the model. For the parameter space studied, web thickness, roller radius and tension were main effects on hinging and web thickness, tape thickness and tension were main effects on peel length. A simplified, conservative model neglecting tension stiffening in the tape span was developed for the purpose of generating a simple criterion for predicting relationships between the variables when hinging would be predicted to no longer occur. This criterion was shown to be conservative, but reasonably accurate for making this prediction. For the parameter space studied, the tensions required to eliminate hinging are relatively high. Experimental data presented correlated reasonably with the model; however, more experimental data should be obtained to better verify the model.

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